Coffeyville Community College

#MATH-120

COURSE SYLLABUS

FOR

CALCULUS WITH ANALYTIC GEOMETRY II

Ryan Willis
Instructor

COURSE NUMBER:	MATH-120 **COURSE TITLE:** Calculus with Analytic Geometry II
CREDIT HOURS:	5
INSTRUCTOR:	R. Willis
OFFICE LOCATION:	Math/Science Office Area, Arts and Sciences Building
PHONE:	Extension 2164
OFFICE HOURS:	See schedule posted on office door
PREREQUISITE(S):	Trigonometry and Calculus with Analytic Geometry I
REQUIRED TEXT AND MATERIALS:	*Calculus* by Larson/Hostetler/Edwards 8th edition

COURSE DESCRIPTION:

This course treats integration techniques, series, polar coordinates and vector applications.

EXPECTED LEARNER OUTCOMES:

1. Apply calculus concepts to trigonometric functions.
2. Apply calculus concepts to the inverse trigonometric functions.
3. Apply calculus concepts to the Hyperbolic Functions
4. Use a variety of methods of integration.
5. Apply the definite integral
6. Use indeterminate forms and improper integrals
7. Work with various series
8. Use Taylor's formula and power series
9. Apply polar coordinates and parametric equations
10. Work with two dimensional vectors
11. Work with three dimensional geometry
12. Work with surfaces
13. Work with vector valued functions

LEARNING TASKS & ACTIVITIES:

Unit I: Trigonometric Functions
Unit II: Inverse Trigonometric Functions; Chapter 5
Unit III: Hyperbolic Functions; Chapter 5
Unit IV: Methods of Integration; Chapter 7/8
Unit V: Application of the Definite Integral; Chapter 7
Unit VI: Indeterminate Forms and Improper Integrals; Chapter 8
Unit VII: Series; Chapter 9

Unit VIII: Taylor’s Formula and Power Series; Chapter 9
Unit IX: Polar Coordinates and Parametric Equations; Chapter 10
Unit X: Two Dimensional Vectors; Chapter 11
Unit XI: Three Dimensional Geometry; Chapter 11
Unit XII: Surfaces; Chapter 11
Unit XIII: Vector Valued Functions; Chapter 12

ASSESSMENT OF OUTCOMES: Grades will be based on quizzes, tests, and homework.

GRADE	%
A	90 - 100
B	80 - 89
C	70 - 79
D	60 - 69
F	< 59

TESTING:

1. Students demonstrate the required behaviors on written examinations over the objectives
2. No "retests" will be given.

HOMEWORK:

1. Homework will be assigned to help students learn the objectives.
2. Homework is due at the time specified by the instructor. Late homework will not be accepted.

ATTENDANCE:

1. Students are expected to attend all class sessions.
2. Work missed due to absences must be made up in advance.
3. Exams missed due to absences may be made up at the discretion of the instructor.
4. Students in attendance are expected to demonstrate appropriate behavior.

COMPETENCIES:

UNIT I: TRIGONOMETRIC FUNCTIONS

APPLY CALCULUS CONCEPTS TO TRIGONOMETRIC FUNCTIONS

1. Sketch the graphs of sin(*x*) and cos(*x*).

2. State $\lim_{x \to 0} \cos(x) = 1$

3. State and prove:

   a) $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

   b) $\lim_{x \to 0} \frac{1-\cos(x)}{x} = 0$

   c) $\frac{d}{dx}(\sin x) = \cos x$

   d) $\frac{df}{dx}(u) = \frac{df}{du}(u)\frac{du}{dx}$ (Chain rule).

   e) $\frac{d}{dx}(cox) = -\sin x$

   f) $\frac{d}{dx}\sin u - \cos u \frac{du}{dx}$

   g) $\frac{d}{dx}\cos u = -\sin u \frac{du}{dx}$

   h) $\frac{d}{dx}\tan u - \sec^2 u \frac{du}{dx}$

   i) $\frac{d}{dx}\cot u = -\csc^2 u \frac{du}{dx}$

   j) $\frac{d}{dx}\sec u - \sec u \tan u \frac{du}{dx}$

   k) $\frac{d}{dx}\csc u = -\csc u \cot u \frac{du}{dx}$

l) $\int \cos x dx = \sin x + c$

m) $\int \sin x dx = -\cos x + c$

n) $\int \sec^2 x dx = \tan x + c$

o) $\int \csc^2 x dx = -\cot x + c$

p) $\int \sec x \tan x dx = \sec x + c$

q) $\int \csc x \cot x dx = -\csc x + c$

4. Use the above rules to take the derivatives or integrals of given functions.
   TEXT: Chapter 2

UNIT II - INVERSE TRIGONOMETRIC FUNCTIONS

APPLY CALCULUS CONCEPTS TO THE INVERSE TRIGONOMETRIC FUNCTIONS

1. Define Inverse Function
2. Define the inverse trigonometric functions.
3. Sketch the graphs of the inverse trigonometric functions.
4. Evaluate the inverse trigonometric functions at given numbers.
5. State and prove the derivative rules and the corresponding integral rules for the inverse trigonometric functions.
6. Use #5 above to take given derivatives and integrals.
   TEXT: 5

UNIT III: HYPERBOLIC FUNCTIONS

APPLY CALCULUS CONCEPTS TO THE HYPERBOLIC FUNCTIONS

1. Define the six hyperbolic functions.
2. Sketch the graphs of the six hyperbolic functions.
3. State and prove:

   a. $\frac{d}{dx}\sinh u - \cosh u \frac{du}{dx}$

   b. $\frac{d}{dx}\cosh u - \sinh u \frac{du}{dx}$

   c. $\cosh^2 u - \sinh^2 u = 1$

d. $\frac{d}{dx}\tanh u = \sec h^2 u \frac{du}{dx}$

e. $\frac{d}{dx}\coth u = -\csc h^2 u \frac{du}{dx}$

f. $\frac{d}{dx}\sec hu = -\sec hu \tanh u \frac{du}{dx}$

g. $\frac{d}{dx}\csc hu = -\csc hu \coth u \frac{du}{dx}$

4. Take given derivatives using #3.
5. State and prove:

a. $\int \cosh u du = \sinh u + c$

b. $\int \sinh u du = \cosh u + c$

c. $\int \sec h^2 u du = \tanh u + c$

d. $\int \csc h^2 u du = -\coth u + c$

e. $\int \sec hu \tanh u du = -\sec hu + c$

f. $\int \csc hu \coth u du = -\csc hu + c$

6. Take given integrals using #5.
7. Define the six inverse hyperbolic functions.
8. Sketch the graphs of the six inverse hyperbolic functions.
9. State and prove:

a. $\frac{d}{dx}\sinh^{-1} u = \frac{u'}{\sqrt{u^2+1}}$

b. $\frac{d}{dx}\cosh^{-1} u = \frac{u'}{\sqrt{u^2-1}}, u > 1$

c. $\frac{d}{dx}\tanh^{-1} u = \frac{u'}{1-u^2}, |u| < 1$

d. $\frac{d}{dx}\coth^{-1}u = \frac{u'}{1-u^2}, |u| > 1$

10. Take given derivatives using #9.
11. State and prove:

a. $\int \frac{du}{\sqrt{u^2+1}} = \sinh^{-1}u + C$

b. $\int \frac{du}{\sqrt{u^2-1}} = \cosh^{-1}u + C, u > 1$

c. $\int \frac{du}{1-u^2} = \tanh^{-1}u + C, |u| < 1$

d. $\int \frac{du}{1-u^2} = \tanh^{-1}u + C, |u| > 1$

TEXT 5

UNIT IV METHODS OF INTEGRATION

USE A VARIETY OF METHODS OF INTEGRATION

1. Given a list of integral rules evaluate $\int f(u)du$ where the proper choice for u gives a problem workable directly from the list.

2. State and prove: $\int udv = uv - \int vdu$

3. Evaluate integrals with terms $a^2 - u^2, a^2 + u^2, u^2 - a^2$ by using trig substitution.

4. Evaluate integrals with terms of the form $ax^2 + bx + c$.

5. Evaluate integrals with rational functions.

6. Use the substitution $z = \tan\frac{x}{2}$ to evaluate integrals containing $\sin x$ and $\cos x$.

   TEXT: 7/8

UNIT V: APPLICATIONS OF THE DEFINITE INTEGRAL

APPLY THE DEFINITE INTEGRAL

1. Find the area between two given curves.

2. Given the velocity of a moving body, find the distance traveled by the body over a given interval of time.
3. Do #2 given the acceleration function.
4. Find the volume of given solids of revolution.
5. Find the arc length of given curves.
6. Find the surface area of given surfaces of revolution.
7. Find moments and centers of mass.
8. Find the force due to hydrostatic pressure.
9. Find work using integrals.

   TEXT Chapter 7

UNIT VI: INDETERMINATE FORMS, IMPROPER INTEGRALS

USE INDETERMINATE FORMSAND IMPROPER INTEGRALS

1. List the symbolic representations of the indeterminate forms.
2. State L'Hospital's Rule.
3. Find limits of indeterminate forms.
4. Evaluate given improper integrals if they converge.

   TEXT 8

UNIT VII: SERIES

WORK WITH VARIOUS SERIES

1. Define sequence.
2. Define limit of a sequence.
3. Define $\lim_{n \to \infty} a_n = \infty$
4. Find the limit of a sequence if it exists.
5. Define: (a) infinite series (b) sum of a series (c) converge and diverge (d) geometric series
6. State the following tests for convergence:
   a. geometric series test
   b. general term test
   c. comparison test
   d. limit
   e. BASIC
   f. p-series
   g. integral test
   h. alternating series test
   i. ratio test
   j. root test
7. Prove that a given series converges or diverges by using the tests in #6.

   TEXT 9

UNIT VIII: TAYLOR'S FORMULA AND POWER SERIES

USE TAYLOR'S FORMULA AND POWER SERIES

1. Write Taylor's Formula with the Remainder for $f(x)$ at *a*.
2. Write Taylor's Formula with the Remainder for a given $f(x)$, *a*, and *n*.
3. Use Taylor's Formula to approximate to a given accuracy.
4. Estimate the error for a given Taylor's Formula approximation.
5. Define power series.
6. for a given power series:
    a. Find its interval of convergence.
    b. Find its radius of convergence.
7. Find a power series representation for a given function and specify its radius of convergence.
8. Use infinite series to approximate given definite integrals to a given degree of accuracy.

<u>TAYLOR'S FORMULA WITH REMAINDER AT a.</u>

$$f(x)=\sum_{k=0}^{n}\frac{f^{(k)}(a)(x=a)^k}{k!}+\frac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1}=P_n(x)+R_n(x),z\in(x,a)$$

Note: 1. $f^{(k)}(a)=\dfrac{d^k f(x)}{dx^k}$ at $x=a$

2. 0! = 1
3. When a = 0 Taylor's Formula is called Maclaurin's Formula.
4. When $R_n(x)$ is "small" $f(x)=P_n(x)$

TEXT 9

UNIT IX: POLAR COORDINATES AND PARAMETRIC EQUATIONS

APPLY POLAR COORDINATES AND PARAMETRIC EQUATIONS

1. Develop the relationship between rectangular coordinates and polar coordinates
2. Find all the ways to name a given polar point.
3. Graph given polar statements.
4. Find the polar coordinates of given rectangular points.
5. Show that a given point is on a given curve.
6. Find the points of intersection of two given polar equations.
7. Change a given polar equation to a rectangular equation.
8. Change a given rectangular equation to polar equation.
9. Fine the angle between the radius vector and the tangent line for a given point on a given curve.
10. Find the areas in given polar regions.
11. Find the arc length of a polar curve.

TEXT Chapter 10

UNIT X: TWO DIMENSIONAL VECTORS

WORK WITH TWO DIMENSIONAL VECTORS

1. Define two dimensional vectors.
2. Give a geometrical interpretation of two dimensional vectors.
3. Define: (a) length of a vector, (b) unit vector (c) Zero vector
4. Define addition of vectors.
5. Give a geometrical interpretation of 4.
6. Define the product of a real number and a vector.
7. Give a geometrical interpretation of 6.
8. Define subtraction of vectors.
9. Give a geometrical interpretation of 9.
10. Prove: The vector from (x_1, y_1) to (x_2, y_2) is $\langle x_2 - x_1, y_2 - y_1 \rangle$
11. Prove: $\frac{<a,b>}{|<a,b>|}$ is a unit vector in the same direction as $<a, b>$
12. Work problems like those on pages 717-719 of the text
13. Define the dot product of two vectors.
14. Prove: $<a, b> \cdot <c, d> = ac + bd$
15. Prove: The projection of V on W is $\frac{V \cdot W}{|W|}$
16. Find the dot product of two vectors.
17. Find the projection of a vector on a vector.
18. Find the angle between two given vectors.
19. Prove: V and W are orthogonal $\Leftrightarrow V \cdot W = 0$

    TEXT 11

UNIT XI: THREE DIMENSIONAL GEOMETRY

WORK WITH THREE DIMENSIONAL GEOMETRY

1. Plot given points.
2. Define three dimensional vector.
3. Give a geometrical interpretation of a three dimensional vector.
4. Define: (a) length of a vector (b) unit vector (c) Zero vector.
5. Define addition of vectors.
6. Give a geometrical interpretation of #5.
7. Define the product of a real number (scalar) and a vector.
8. Give a geometrical interpretation of #7.
9. Prove: The vector from (x_1, y_1, z_1) to (x_2, y_2, z_2) is $\langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$
10. Define subtraction of vectors.
11. Give a geometrical interpretation of 10.
12. Prove: $\frac{<a,b,c>}{|<a,b,c,>|}$ is a unit vector in the same direction as $<a,b,c>$.
13. Work problems like those on pages 725-727 of the text.

14. Prove: the distance between (x_1, y_1, z_1) and (x_2, y_2, z_2)
is $(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$.
15. Describe geometrically a set of points for which information is given.
16. Write the equation for a given set of points.
17. Define: (a) direction angles (b) direction cosines (c) direction numbers.
18. Find the direction angles, cosines, and numbers for a given line.
19. Find the angle between two given lines.
20. Work problems like those on pages 735-736.
21. Define the dot produce of two vectors.
22. Prove: $\langle a_1, a_2, a_3 \rangle \cdot \langle b_1, b_2, b_3 \rangle = a_1 b_1 + a_2 b_2 + a_3 b_3$
23. Prove: The projection of V on W is $\frac{V \cdot W}{|W|}$
24. Find the dot product of two vectors.
25. Find the projection of a vector on another given vector.
26. Find the angle between two given vectors.
27. Prove: V and W are orthogonal $\Leftrightarrow V \cdot W = 0$
28. Write the equation of a line given information about it.
29. Determine if two given lines are perpendicular.
30. Define the cross product of two vectors.
31. State: $< a_1, a_2, a_3 > X < b_1, b_2, b_3 > = < a_1, a_2, a_3 > x < b_1, b_2, b_3 > = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
32. Find the cross product of given vectors.
33. Write the equation of a plane for which information is given.
34. Find the angle between two given planes.
35. Find the distance between a point and a plane.
36. Sketch given lines or planes.
37. Prove: The area of the parallelogram determined by A and B is $|AxB|$.
38. Find the area of the parallelogram determined by two given vectors.
39. Prove: The volume of the parallelepiped determined by vectors, A B and C is $|A \cdot BxC|$.
40. Find the volume of a parallelepiped determined by three given vectors.
TEXT 11

UNIT XII: SURFACES

WORK WITH SURFACES

1. Write the equation of a sphere given information about it.
2. Find the center and radius of a sphere given information about it.
3. Define cylinder.
4. Describe and sketch a cylinder whose equation is given.
5. Name and sketch a quadratic surface whose equation is given.
6. Write the relationship among cartesian, cylindrical, and spherical coordinates.

7. Given a point in one coordinate system, write the point in the two other coordinate systems.
8. Given an equation in one coordinate system, write the equation in the other two coordinate systems.

   TEXT 11

UNIT XIII

WORK WITH VECTOR VALUED FUNCTIONS

1. Define vector valued function.
2. Graph the range of given vector valued functions.
3. Define $r'(t)$ for a given vector valued function r(*t*).
4. Find r' and r'' for a given *r*.
5. For a given $r(t)$ and *c* find $r'(c), r''(c)$. Plot $r(c)$ and sketch $r'(c)$ and $r''(c)$ at $r(c)$.
6. For a given $r(t)$ find the *x*-*y* equation a portion of whose graph is the graph of the range of $r(t)$.
7. Find the length of a given curve.
8. For a given $r(t)$ find: $v, a, |v|, T, k, |k|, N, B, R$, osculating plane, normal and tangential components of a.
9. Graph all the vectors in #8 at a given point.

   TEXT Chapter 12

This syllabus is subject to revision with prior notification to the student by the instructor.