

**Coffeyville Community College**

**MATH-115**

**COURSE SYLLABUS**

**FOR**

**CALCULUS WITH ANALYTIC  
GEOMETRY I**

**Ryan Willis  
Instructor**

**COURSE NUMBER:** MATH-115 **COURSE TITLE:** Calculus With Analytic Geometry I

**CREDIT HOURS:** 5

**INSTRUCTOR:** Ryan Willis

**OFFICE LOCATION:** Math/Science Office Area, Arts & Sciences Building

**OFFICE HOURS:** See posted schedule

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**PREREQUISITE(S):** Trigonometry

**REQUIRED TEXT AND MATERIALS:** *Calculus* by Larson/Hostetler/Edwards

**COURSE DESCRIPTION:** The course is a study of derivatives and integrals with applications.

**EXPECTED LEARNER OUTCOMES:**

1. The student will understand the rectangular coordinate system.
2. The student will understand second degree equations.
3. The student will define and use the concept of functions.
4. The student will define and use the concept of limit.
5. The student will define and use the concept of continuity.
6. The student will define and use the concept of derivative.
7. The student will apply the concept of derivative to sketch curves.
8. The student will use application of the concept of derivative.
9. The student will develop and use Newton's Method.
10. The student will define and use the concept of indefinite integral.
11. The student will define and use the concept of definite integral.
12. The student will approximate definite integrals by numerical methods.
13. The student will define and use exponential and logarithmic functions.
14. The student will define and use the trigonometric functions.

15. The student will define and use the inverse trigonometric functions.

**LEARNING TASKS  
& ACTIVITIES:**

Employing the concepts of set, real number, and function (as found in College Algebra and Trigonometry courses) the first course in the 13 hour sequence develops the concepts of limit, and the related concepts of derivative and integral.

- Unit I: Introduction, Chapter 1:27-36
- Unit II: Analytic Geometry, Chapter 1:14-18; 10:627-659
- Unit III: Functions, Chapter 1:36-47
- Unit IV: Development of Limit, Chapter 2:65-75, 92-93, 101-109; 4:206-214
- Unit V: Continuity, Chapter 2:75-100
- Unit VI: Development of the Derivative, Chapter 3:111-120, 129-164
- Unit VII: Curve Sketching, Chapter 4:177-206, 215-233
- Unit VIII: Applications of Derivative, Chapter 3:121-129, 164-175; 4:241-248
- Unit IX: Newton's Method, Chapter 4:233-240
- Unit X: Development of the Indefinite Integral, Chapter 5:259-269
- Unit XI: Fundamental Theorem of Calculus, Chapter 5:269-300
- Unit XII: Numerical Integration, Chapter 5:312-320
- Unit XIII: Exponential and Logarithmic Functions, Chapter 6:325-364
- Unit XIV: Trigonometric Functions, Chapter 1:47-60; 2:86-90; 3:134-138, 143-147, 153-156; 5:262-267
- Unit XV: Inverse Trig Functions, Chapter 6:371-383

**ASSESSMENT OF  
OUTCOMES:**

The semester grade will be determined by tests, quizzes, and homework as shown in the following table.

- A = 90-100%
- B = 80- 89%
- C = 70- 79%
- D = 60- 69%
- F = 0- 59%

**TESTING:**

1. Students demonstrate the required behaviors on written exams over the objectives.
2. Exams will be taken at the time specified by the instructor.

**HOMEWORK:**

1. Homework will be assigned to help students meet the objectives.
2. Homework is due at the time specified by the instructor. Late homework will not be accepted.

**ATTENDANCE:**

Students are expected to attend all classes.

## COMPETENCIES:

### UNIT I: INTRODUCTION

The student will understand the rectangular coordinate system.

1. Plot any ordered pair on a rectangular coordinate system.
2. Develop the distance formula between two points.
3. Find the distance between any two points.
4. Define slope of a straight line.
5. Find the slope of a line given:
  - a. two of its points
  - b. its equation
6. Write the equation of a line given:
  - a. two of its points
  - b. one of its points and its slope
7. Define:
  - a. Parallel lines
  - b. Perpendicular lines
8. Prove: if  $Ax + By + C = 0$  is the equation of a line, then its slope is  $-A/B$  and its y-intercept is  $-C/B$ .
9. Graph straight lines given:
  - a. their equations
  - b. one of their points and their slopes

TEXT: Chapter 1:27-36

### UNIT II: ANALYTIC GEOMETRY

The student will understand second degree equations.

1. For a given second degree equation:
  - a. determine what type of curve it is
  - b. for circles, find the center and radius
  - c. for parabolas, find center, focus, vertex, and directrix
  - d. for an ellipse, find center, foci, and vertices
  - e. for a hyperbola, find center, foci, vertices, and asymptotes
  - f. graph the equation and label the parts found above
2. Write the equation of:
  - a. a circle
  - b. parabola
  - c. an ellipse
  - d. a hyperbola

TEXT: 1:14-18; 10:627-659

### UNIT III: FUNCTIONS

The student will define and use the concept of function.

1. Define:
  - a. function
  - b. domain
  - c. range
2. For a given function:
  - a. graph it
  - b. determine its range and domain

TEXT: 1:36-47

### UNIT IV: DEVELOPMENT OF LIMIT

The student will define and use the concept of limit.

1. Define  $\lim_{x \rightarrow c} f(x) = L$  and show each part geometrically
2. State and use:
  - a. if  $a < b$  then  $a + c < b + c$
  - b. if  $ab > 0$  and  $a < b$  then  $1/b < 1/a$
  - c. if  $a < 0 < b$  then  $1/a < 0 < 1/b$
  - d. if  $a < b$  and  $c > 0$  then  $a < bc$
  - e. if  $a < b$  and  $c < 0$  then  $ac > bc$
  - f.  $x^a \neq a^x$
  - g.  $x^a \neq a^x$  or  $x^{-a}$
3. Check out a library book that has the definition of limit of a function. Bring it to class and point out the definition to the instructor.
4. Find the limit of a function and prove it using the definition of limit.
5. Define  $\lim_{x \rightarrow c} f(x) = k$  where  $c$  and  $k$  are any of the following:  
 $c$  by  $4, -4, \text{finite } c, c^+, c^-$        $k$  by  $4, -4, \text{finite } k$

TEXT: 2:65-75, 92-93, 101-109; 4: 206-214

## UNIT V: CONTINUITY

The student will define and use the concept of continuity.

1. State:
  - a.  $\lim_{x \rightarrow a} (mx + b) = ma + b \forall m, a, x, b$
  - b.  $\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$
  - c.  $\lim_{x \rightarrow a} kf(x) = k \lim_{x \rightarrow a} f(x)$
  - d.  $\lim_{x \rightarrow a} f(x)g(x) = \lim_{x \rightarrow a} f(x) \lim_{x \rightarrow a} g(x)$
  - e.  $\lim_{x \rightarrow a} f(x)/g(x) = \lim_{x \rightarrow a} f(x) / \lim_{x \rightarrow a} g(x)$
2. State and prove:
  - a.  $\lim_{x \rightarrow c} x^n = c^n, n \in \mathbb{N}$
  - b. If  $f(x)$  and  $g(x)$  are polynomials, then  $\lim_{x \rightarrow c} f(x)/g(x) = f(c)/g(c), g(c) \neq 0$
3. Determine the limit of given rational expressions.
4. Define  $f(x)$  is continuous at  $x=c$ .
5. Explain how the rigorous definition in 4 above is consistent with the "definition" that  $f(x)$  is continuous at  $x=c$  the curve has no holes or breaks at  $x=c$ .
6. State and prove that every rational function is continuous at every point in its domain.

TEXT: 2:75-100

## UNIT VI: DEVELOPMENT OF THE DERIVATIVE

The student will define and use the concept of derivative.

1. Define:  $f'(c)$ , the derivative of  $f(x)$  of the derivative at  $x=c$
2. Show each part of the definition of derivative geometrically.

3. State the geometrical significance of  $f'(c)$  and support your answer.
4. Find the derivative of given functions using only the definition.
5. State and prove: If  $f'(c)$  is finite then  $f(x)$  is continuous at  $x=c$ .
6. State and prove:

a. 
$$\frac{dk}{dx} = 0$$

b. 
$$\frac{dx^n}{dx} = nx^{n-1} \text{ where } n \in \mathbb{N}$$

c. 
$$\frac{dkf(x)}{dx} = k \frac{df(x)}{dx}$$

d. 
$$\frac{d[f(x) + g(x)]}{dx} = \frac{df(x)}{dx} + \frac{dg(x)}{dx}$$

e. 
$$\frac{df(x)g(x)}{dx} = f(x) \frac{dg(x)}{dx} + g(x) \frac{df(x)}{dx}$$

f. 
$$\frac{df^n(x)}{dx} = nf^{n-1} \frac{df(x)}{dx}$$

g. 
$$\frac{df(x)/g(x)}{dx} = \frac{\frac{g(x)df(x)}{dx} - \frac{f(x)dg(x)}{dx}}{g(x)^2}$$

h. 
$$\frac{df^n(x)}{dx} = nf^{n-1}(x) \frac{df(x)}{dx} \text{ where } n \in \mathbb{I}$$

i. 
$$\frac{df^n(x)}{dx} = nf^{n-1}(x) \frac{df(x)}{dx} \text{ where } n \in \mathbb{Q}$$

7. Find the derivative of given functions.
8. Find  $\frac{d^n y}{dx^n}$  for given  $y$  where  $n \in \mathbb{N}$
9. Find the tangent line to a given curve at a given point.

TEXT: 3:111-120, 129-164

## UNIT VII: CURVE SKETCHING

The student will apply the concept of derivative to sketch curves.

1. Define:
  - a. Increasing Function
  - b. Decreasing Function
2. Explain what is happening to  $f(x)$  at  $c$  such that:
  - a.  $f'(c) = 0$
  - b.  $f'(c) > 0$
  - c.  $f'(c) < 0$
  - d.  $f'(c) DNE$
3. Given a function  $f(x)$ :
  - a. Find the points at which  $f(x)$ :
    - (i) has horizontal tangents
    - (ii) is increasing
    - (iii) is decreasing
    - (iv) has corners
    - (v) is discontinuous
    - (vi) has end points
  - b. graph it showing the features above.
4. Explain what is happening to  $f(x)$  at  $c$  such that:
  - a.  $f'(c) > 0$
  - b.  $f'(c) < 0$
  - c.  $f'(c) = 0$
  - d.  $f'(c) DNE$
5. Find the points at which a given  $f(x)$ :
  - a. is concave up
  - b. is concave down
  - c. has inflection points
  - d. has relative extrema
  - e. has absolute extrema
6. Work maximum-minimum problems.

TEXT: 4:177-206, 215-233

## UNIT VIII: APPLICATIONS OF DERIVATIVE

The student will use application of the concept of derivative.

1. Explain why  $\frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$  for “small”  $\Delta x$ .
2. Approximate  $f(x + \Delta x)$  for given  $f, x$ , and  $\Delta x$ .

3. Approximate  $\Delta y$  for given  $f$ ,  $x$  and  $\Delta x$ .
4. Define  $dy$  and  $dx$ .
5. Find  $dy$  for a given  $y$ .
6. Explain why  $\Delta y \approx dy$  for “small”  $\Delta x$ .
7. State that  $\frac{dy}{dx}$  is the instantaneous rate of change of  $y$  with respect to  $x$ .
8. Work related rate problems.

TEXT: 3:121-129, 164-175; 4:241-248

#### UNIT IX: NEWTON'S METHOD

The student will develop and use Newton's Method.

1. Develop Newton's Method.
2. For Newton's Method of approximating list the:
  - a. necessary conditions
  - b. procedure
  - c. rationale
  - d. error
3. Use Newton's Method to approximate the zero of a given function to a given degree of accuracy.
4. Give the upper bound on the error when using Newton's Method.

#### NEWTON'S METHOD OF APPROXIMATING $r \in [a, b] \ni f(r) = 0$

#### NECESSARY CONDITIONS

OVER  $[a, b]$ ,  $f(a)f(b) < 0$

$f(x)$  exists  $\forall x$

$f'(x) \neq 0 \forall x$

#### PROCEDURE

1. Let  $x_1 = a$
2. Let  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

3. When  $x_n$  approaches a limit, then  $x_n$  is the desired approximation.

#### RATIONALE

1.  $x_{n+1}$  is the  $x$ -intercept of the tangent line to  $y = f(x)$  at  $(x_n, f(x_n))$
2. The tangent line is an approximation of the curve "near" the point of tangency.

#### ERROR

$$|x_{n+1} - r| \leq \frac{f(x_n)}{f'(x_n)}$$

TEXT: 4:233-240

#### UNIT X: DEVELOPMENT OF THE INDEFINITE INTEGRAL

The student will define and use the concept of indefinite integral.

1. Define  $\int f(x)dx$
2. State and prove:
  - a.  $\int du = u + c$
  - b.  $\int Kf(x)dx = K \int f(x)dx + c$
  - c.  $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx + c$
  - d.  $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1 \in Q$
3. Take the integral of given functions.

TEXT: 5:259-269

#### UNIT XI: FUNDAMENTAL THEOREM OF CALCULUS

The student will define and use the concept of definite integral.

1. Define  $\sum_{i=1}^N a_i$

2. State and prove:

a. 
$$\sum_{i=1}^N ca_i = c \sum_{i=1}^N a_i$$

b. 
$$\sum_{i=1}^N (a_i + b_i) = \sum_{i=1}^N a_i + \sum_{i=1}^N b_i$$

c. 
$$\sum_{i=1}^N c = nc$$

d. 
$$\sum_{i=1}^N i = n(n+1)/2$$

e. 
$$\sum_{i=1}^N i^2 = n(n+1)(2n+1)/6$$

3. a. Define the area,  $A_a b$ , determined by  $y=f(x)$ ,  $x=0$ ,  $x=a$ ,  $x=b$ , and the  $x$ -axis.

b. Explain this definition.

4. Find the area of a given region by using 3(a).

5. State: If  $m$  and  $M$  are the minimum and maximum values of a continuous  $f(x)$  over  $[a,b]$  then  $\forall m \leq y \leq M \exists x \in [a,b] \ni f(x) = y$ .

6. State and prove the Fundamental Theorem of Calculus.

7. Find the area determined by a given  $f(x)$ ,  $x=a$ ,  $x=b$ , and the  $x$ -axis.

8. State 
$$\lim_{N \rightarrow \infty} \sum_{i=1}^N f(x) \Delta x = \int_a^b f(x) dx$$

9. Find  $\int_a^b f(s) dx$

10. Define definite integral.

TEXT: 5:269-300

## UNIT XII: NUMERICAL INTEGRATION

The student will approximate definite integrals by numerical methods.

1. Write the approximation formula for:

a. Definition of  $A^b$

- b. Trapezoidal Rule
  - c. Simpson's Rule
2. Write the error upper bound formula for:
  - a. Trapezoidal Rule
  - b. Simpson's Rule
3. Approximate given integrals using:
  - a. Definition of  $A^b$
  - b. Trapezoidal Rule
  - c. Simpson's Rule
4. Find the upper bound on the error for a given  $f(x), a, b$ , and  $n$  using:
  - a. Trapezoidal Rule
  - b. Simpson's Rule
5. Find an  $n$  such that the error is less than a given number for a given  $f(x), a$ , and  $b$  using:
  - a. Trapezoidal Rule
  - b. Simpson's Rule

TEXT: 5:312-320

### UNIT XIII: EXPONENTIAL AND LOGARITHMIC FUNCTIONS

The student will define and use exponential and logarithmic functions.

1. State and prove:
  - a. 
$$\frac{df(u)}{dx} = \frac{df(u)}{du} \cdot \frac{du}{dx}$$
  - b. 
$$\frac{d}{dx} \int f(t) dt = f(x)$$
2. Define:
  - a.  $\ln(x)$
  - b. 
$$\frac{d \ln|u|}{dx} = \frac{1}{u} \cdot \frac{du}{dx}$$
 (state and prove)
3. State and prove
  - a.  $\ln xy = \ln x + \ln y$
  - b.  $\ln x/y = \ln x - \ln y$
  - c.  $\ln x^r = r \ln x$
4. Sketch  $y = \ln x$

5. Define:
- $\exp(x)$
  - $e$
6. State and prove:
- $e^x = \exp(x)$
  - $\frac{de^u}{dx} = e^u \frac{du}{dx}$
7. State and prove:
- $\int \frac{du}{u} = \ln|u| + c$
  - $\int e^u du = e^u + c$
8. a. Define:  $a^x$
- b. Prove  $\log_b x = \frac{\ln x}{\ln b}$
9. State and prove
- $\frac{da^u}{dx} = a^u \ln a \frac{du}{dx}$
  - $\int a^u du = \frac{1}{\ln a} a^u + c$
10. Define  $\log_b x$
11. State and prove:
- $\frac{d}{dx} \log_a |u| = \frac{1}{u \ln a} \cdot \frac{du}{dx}$
  - $\log_e u = \ln u$

12. Sketch:

a.  $y = \log_b x$

b.  $y = a^x$

13. Find integrals or derivatives of given functions using the rules above.

TEXT: 6:325-364

#### UNIT XIV: TRIGONOMETRIC FUNCTIONS

The student will define and use the trigonometric functions.

1. Sketch the graphs of  $\sin(x)$  and  $\cos(x)$

2. State:  $\lim_{x \rightarrow 0} \cos(x) = 1$

3. State and prove:

a.  $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$

b.  $\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x} = 0$

c.  $\frac{d}{dx}(\sin x) = \cos x$

d.  $\frac{df}{dx}(u) = \frac{df}{du}(u) \frac{du}{dx}$  (chain rule)

e.  $\frac{d}{dx}(\cos) = -\sin x$

f.  $\frac{d}{dx} \sin u = \cos u \frac{du}{dx}$

g.  $\frac{d}{dx} \cos u = -\sin u \frac{du}{dx}$

h.  $\frac{d}{dx} \tan u = \sec^2 u \frac{du}{dx}$

i.  $\frac{d}{dx} \cot u = -\csc^2 u \frac{du}{dx}$

j.  $\frac{d}{dx} \sec u = \sec u \tan u \frac{du}{dx}$

k.  $\frac{d}{dx} \csc u = -\csc u \cot u \frac{du}{dx}$

l.  $\int \cos x dx = \sin x + c$

m.  $\int \sin x dx = -\cos x + c$

n.  $\int \sec^2 x dx = \tan x + c$

o.  $\int \csc^2 x dx = -\cot x + c$

p.  $\int \sec x \tan x dx = \sec x + c$

q.  $\int \csc x \cot x dx = -\csc x + c$

4. Use the above rules to take the derivatives or integrals of given functions.

TEXT: 1:47-60; 2:86-90; 3:134-138, 143-147, 153-156; 5:262-267

#### UNIT XV: INVERSE TRIG FUNCTIONS

The student will define and use the inverse trigonometric functions.

1. Define Inverse Function.
2. Define the inverse trigonometric functions.
3. Sketch the graphs of the inverse trigonometric functions.
4. Evaluate the inverse trigonometric functions at given numbers.
5. State and prove the derivative rules for the inverse trigonometric functions.
6. Take derivatives of given inverse trigonometric functions.

TEXT: 6:371-383

**This syllabus is subject to revision with prior notification to the student by the instructor.**